



第15讲：三角恒等变换题型拓展及方法归纳(1)

题型一：和差公式

1. 证明： $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

2. $\sin \frac{\pi}{12} =$ _____;

$\sin \frac{5\pi}{12} =$ _____;

$\tan \frac{\pi}{12} =$ _____;

$\tan \frac{5\pi}{12} =$ _____.

3. (2018 全国卷 II) 已知 $\sin\alpha + \cos\beta = 1$, $\cos\alpha + \sin\beta = 0$, 则 $\sin(\alpha + \beta) =$ _____.

4. (2024·甲卷) 已知 $\frac{\cos\alpha}{\cos\alpha - \sin\alpha} = \sqrt{3}$, 则 $\tan\left(\alpha + \frac{\pi}{4}\right) =$ ()

A. $2\sqrt{3} + 1$

B. $2\sqrt{3} - 1$

C. $\frac{\sqrt{3}}{2}$

D. $1 - \sqrt{3}$

5. (2024·新高考 I) 已知 $\cos(\alpha + \beta) = m$, $\tan\alpha\tan\beta = 2$, 则 $\cos(\alpha - \beta) =$ ()

A. $-3m$

B. $-\frac{m}{3}$

C. $\frac{m}{3}$

D. $3m$





6. (2024·新高考Ⅱ) 已知 α 为第一象限角, β 为第三象限角, $\tan\alpha + \tan\beta = 4$, $\tan\alpha\tan\beta = \sqrt{2} + 1$, 则 $\sin(\alpha + \beta) =$ _____.

7. (2022·新高考Ⅱ) 若 $\sin(\alpha + \beta) + \cos(\alpha + \beta) = 2\sqrt{2}\cos(\alpha + \frac{\pi}{4})\sin\beta$, 则 ()

- A. $\tan(\alpha - \beta) = 1$ B. $\tan(\alpha + \beta) = 1$ C. $\tan(\alpha - \beta) = -1$ D. $\tan(\alpha + \beta) = -1$

8. 已知 $\sin x - \sin y = -\frac{2}{3}$, $\cos x - \cos y = \frac{2}{3}$, 且 x, y 为锐角, 则 $\tan(x - y) =$ ()

- A. $\frac{2\sqrt{14}}{5}$ B. $-\frac{2\sqrt{14}}{5}$ C. $\pm\frac{2\sqrt{14}}{5}$ D. $\pm\frac{5\sqrt{14}}{28}$

9. 若 $0 < \alpha < \frac{\pi}{2}$, $-\frac{\pi}{2} < \beta < 0$, $\cos(\frac{\pi}{4} + \alpha) = \frac{1}{3}$, $\cos(\frac{\pi}{4} - \frac{\beta}{2}) = \frac{\sqrt{3}}{3}$, 则 $\cos(\alpha + \frac{\beta}{2}) =$ ()

- A. $\frac{\sqrt{3}}{3}$ B. $-\frac{\sqrt{3}}{3}$ C. $\frac{5\sqrt{3}}{9}$ D. $-\frac{\sqrt{6}}{9}$

10. 若 $\sin(\frac{\pi}{6} - \alpha) = \frac{1}{3}$, 则 $\cos(\frac{2\pi}{3} + 2\alpha) =$ ()

- A. $-\frac{7}{9}$ B. $-\frac{1}{3}$ C. $\frac{1}{3}$ D. $\frac{7}{9}$

11. (2012 江苏) 设 α 为锐角, 若 $\cos(\alpha + \frac{\pi}{6}) = \frac{4}{5}$, 则 $\sin(2\alpha + \frac{\pi}{12})$ 的值为 _____.





12. 已知 α, β 均为锐角, 且 $\cos(\alpha + \beta) = \frac{\sin \alpha}{\sin \beta}$, 则 $\tan \alpha$ 的最大值是 ()

- A. $\frac{\sqrt{2}}{5}$ B. $\frac{\sqrt{2}}{4}$ C. 2 D. 4

13. 已知两个锐角 α, β , 且 $\sin \alpha = \frac{\sqrt{5}}{5}, \cos \beta = \frac{3\sqrt{10}}{10}$, 则 $\alpha + \beta =$ _____.

14. 已知 $\alpha, \beta \in (0, \pi)$, 且 $\tan(\alpha - \beta) = \frac{1}{2}, \tan \beta = -\frac{1}{7}$, 则 $2\alpha - \beta =$ _____.

15. 证明: (1) $\sin(\alpha - \beta) \sin(\alpha + \beta) = \sin^2 \alpha - \sin^2 \beta$;
(2) $\cos(\alpha - \beta) \cos(\alpha + \beta) = \cos^2 \alpha - \sin^2 \beta$.

题型二: 二倍角及降幂公式

16. 化简: $\sqrt{1 - \sin 8} =$ _____.

17. (2012 山东) 若 $\theta \in [\frac{\pi}{4}, \frac{\pi}{2}]$, $\sin 2\theta = \frac{3\sqrt{7}}{8}$, 则 $\sin \theta =$ ()

- A. $\frac{3}{5}$ B. $\frac{4}{5}$ C. $\frac{\sqrt{7}}{4}$ D. $\frac{3}{4}$





18. $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \underline{\hspace{2cm}}$.

19. $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \underline{\hspace{2cm}}$.

20. (2023·新高考 II) 已知 α 为锐角, $\cos \alpha = \frac{1+\sqrt{5}}{4}$, 则 $\sin \frac{\alpha}{2} = (\quad)$

A. $\frac{3-\sqrt{5}}{8}$

B. $\frac{-1+\sqrt{5}}{8}$

C. $\frac{3-\sqrt{5}}{4}$

D. $\frac{-1+\sqrt{5}}{4}$

21. (2023·新高考 I) 已知 $\sin(\alpha - \beta) = \frac{1}{3}$, $\cos \alpha \sin \beta = \frac{1}{6}$, 则 $\cos(2\alpha + 2\beta) = (\quad)$

A. $\frac{7}{9}$

B. $\frac{1}{9}$

C. $-\frac{1}{9}$

D. $-\frac{7}{9}$

22. (2019 全国 II) 已知 $\alpha \in (0, \frac{\pi}{2})$, $2\sin 2\alpha = \cos 2\alpha + 1$, 则 $\sin \alpha = (\quad)$

A. $\frac{1}{5}$

B. $\frac{\sqrt{5}}{5}$

C. $\frac{\sqrt{3}}{3}$

D. $\frac{2\sqrt{5}}{5}$

23. 已 $\tan \frac{\theta}{2} = \frac{\sqrt{5}-1}{2}$, 则 $\frac{\sin 2\theta}{1-\cos 2\theta} = (\quad)$

A. $\frac{1}{2}$

B. $\frac{\sqrt{5}+1}{2}$

C. 2

D. $\sqrt{5}+1$





24. 化简 $\frac{(1 + \sin\theta + \cos\theta)(\sin\frac{\theta}{2} - \cos\frac{\theta}{2})}{\sqrt{2} + 2\cos\theta} (0 < \theta < \pi)$

25. 化简 $\frac{\cos 40^\circ}{\cos 25^\circ \sqrt{1 - \sin 40^\circ}}$

26. 已知 $\sin\alpha = \frac{1}{2} + \cos\alpha$, $\alpha \in (0, \frac{\pi}{2})$, 则 $\frac{\cos 2\alpha}{\sin(\alpha - \frac{\pi}{4})} =$ _____.







练习

1. 已知 $\sin\left(\alpha + \frac{\pi}{3}\right) + \sin\alpha = \frac{4\sqrt{3}}{5}$, 则 $\sin\left(\alpha + \frac{7\pi}{6}\right) = \underline{\hspace{2cm}}$.
2. 若 $6\sin^2\alpha + \sin\alpha\cos\alpha - 2\cos^2\alpha = 0, \alpha \in \left[\frac{\pi}{2}, \pi\right)$, 则 $\sin\left(2\alpha + \frac{\pi}{3}\right) = \underline{\hspace{2cm}}$.
3. 已知 $\tan\alpha = \frac{1}{2}$, 则 $\cos\left(2\alpha + \frac{\pi}{2}\right) = \underline{\hspace{2cm}}$.

